About the distribution of forces in permanent magnets

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Abstract – **Different methods can be proposed to determine the global force on Magnets (equivalent models, Maxwell stress tensor, virtual work principle...). The same value is obtained whatever the method while the local force distribution differs. This problem (i.e. different local force distributions on permanent magnets) has not been solved yet. Though, the determination of the physical distribution of forces is a crucial point, both for practical applications and for theory. Thus, the different methods allowing the calculation of the local force distribution in permanent magnets are described and compared. Evidences are given that only the method based on the virtual work principle provides reliable results.**

Index terms: **permanent magnet, magnetic force densities, magnetic charge, magnetising current, Maxwell stress tensor, virtual work.**

I. INTRODUCTION

The calculation of forces is essential for the design of electromagnetic devices. The determination of global force as well as local force distribution (or density) is essential for the design of the system. Thus, the use of a correct method to calculate theses quantities is required. Actual computations and comparisons are done with the finite element method (FEM), which provides an approximated solution for the system [1][2]. The different methods used for the magnetic force calculation are either based on mathematical models (such as equivalent source and Maxwell stress tensor) or on the physical principle of the virtual work [2].

Theoretically, all the methods give the same result for the global force calculation. Though, the accuracy depends on the precision of the FEM magnetic solution, as well as on the methods themselves. Some of these methods lead to a rather poor precision even for fine meshes [3].

Concerning the local distribution, each method provides a specific and different result for the force distribution on rigid permanent magnets (the magnetisation $\boldsymbol{M}$ is constant all over the magnet). These methods are discussed and compared. Evidences are given that:

- the methods based on mathematical approaches are only globally equivalent to the initial system and do not represent the actual distribution of forces,
- only the method based on the virtual work principle has a physical meaning [2][3].

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II. DESCRIPTION OF THE METHODS

Different methods can be used for the calculation of the electromagnetic force densities in permanent magnets. These different methods either based on equivalent source models, on the Maxwell stress tensor or on the virtual work principle are described:

A. Equivalent source models

The methods based on the equivalent source models either rely on the magnetic charges or on the magnetising currents. These methods consist in replacing the permanent magnet by an equivalent volume and surface distribution of current or magnetic charges [3][6].

1) Equivalent magnetic charges: The magnet is replaced by a volume charge density ρ_V and by a surface charge density ρ_S defined as [3][6]:

$$\rho_V = -\mu_0 \, div\, \boldsymbol{M} \quad (1)$$

$$\rho_S = \mu_0 \, (\boldsymbol{n} \cdot \boldsymbol{M}) \quad (2)$$

Where $\boldsymbol{M}$ is the magnetisation given in terms of the remanent induction $\boldsymbol{B_r}$ by $\boldsymbol{M} = \boldsymbol{B_r}/\mu_0$, $\boldsymbol{n}$ is the unit vector normal to the surface of the magnet and μ_0 is the permeability of the vacuum.

For constant magnetisation $\boldsymbol{M}$, the volume charge density is zero. The resulting force density is only on the surface of the magnet and is given by:

$$\boldsymbol{f_s} = \rho_S \boldsymbol{H_s} \quad (3)$$

Where $\boldsymbol{H_s}$ is the average of the magnetic field on the surface of the magnet defined as $\boldsymbol{H_s} = (\boldsymbol{H^+} + \boldsymbol{H^-})/2$ [3][6].

2) Equivalent magnetising current: In this method the magnet is replaced by a nonmagnetic material with volume current density $\boldsymbol{J_v}$ and surface current density $\boldsymbol{J_s}$. These current densities are defined as [3][6]:

$$\boldsymbol{J_v} = \boldsymbol{rot\, M} \quad (4)$$

$$\boldsymbol{J_s} = -\,\boldsymbol{n} \times \boldsymbol{M} \quad (5)$$

For constant magnetisation, the volume current density vanishes and the resulting surface force density is given by Lorentz's force [3][6] :

$$\boldsymbol{f_s} = \boldsymbol{J_s} \times \boldsymbol{B_s} \quad (6)$$

Where $\boldsymbol{B_s}$ is the average of the magnetic induction on the surface of the magnet given by $\boldsymbol{B_s} = (\boldsymbol{B}^+ + \boldsymbol{B}^-)/2$ [3][6].

The magnetic global force is given by the integration of (3) or (6) on the surface of the magnet. As these methods are based on equivalent models, there is no reason for which the resulting force densities should have any physical reality. They are only globally equivalent to the original magnet. Therefore, one must not use the force distributions resulting from the use of the equivalent models.

B. Maxwell stress tensor method

For the calculation of forces by the Maxwell stress tensor, one can use either a surface integration or a volume integration [1][3][8]. Each way presents some advantages with respect to the other. These advantages are discussed hereafter.

1) Surface integration: The force on the magnet can be calculated by the integration of the Maxwell stress tensor on a surface surrounding the magnet [3][6]. The terms of the Maxwell stress tensor are defined as:

$$\tau_{ij} = \frac{1}{\mu_0} B_i B_j - \delta_{ij} \frac{1}{2\mu_0} B^2 \tag{7}$$

Where δ_{ij} is the Kronecker delta. The force densities are then calculated by [1][3]:

$$\boldsymbol{f} = \frac{1}{\mu_0}(\boldsymbol{B} \cdot \boldsymbol{n})\boldsymbol{B} - \frac{1}{2\mu_0} B^2 \cdot \boldsymbol{n} \tag{8}$$

The global force is calculated by the integration of (8) on an arbitrary surface S surrounding the magnet. Theoretically, the choice of this surface should not influence the result of the force calculation. Though, as the force is obtained with an approximate finite element solution, the result depends on the choice of this surface [1][3][7].

To overcome the definition of the surface of integration, a volume integration method is proposed by [8]. The advantage of this method is that the same FEM mesh is used to calculate the force and the electromagnetic computation.

2) Volume integration: With this method, the force on the direction i on a node k of the mesh is given by:

$$f_{ik} = -\int_V \tau_{ij} \partial_j \alpha_k \, dV \tag{9}$$

Where α_k are the nodal shape functions [3][8]. The force on the node is given by the summation of the contribution of all elements surrounding the node. The global force is the summation of the forces on all the nodes of the magnet. This method is straightforward in a FEM code as the same FEM mesh is used.

The methods based on the Maxwell stress tensor rely on a mathematical artifice. As the equivalent source models, the force given by the Maxwell stress tensor has only a global interpretation. The resulting force densities are not physical. In principle, none of these distributions can be assumed to be the actual physical distribution of forces [2][3].

A method presented by [2] and based on the physical principle of the virtual work is described hereafter. Thus, one can expect the resulting force densities to have a physical meaning. After some considerations, they can be assumed as the actual distribution of forces on the magnet [2].

C. Virtual work method

By the virtual work method, the magnetic force is given by the derivation of the co-energy ($W'=W'(\boldsymbol{H})$) or energy ($W=W(\boldsymbol{B})$) with respect to a virtual displacement [1][2][4]. According to the formulation used to solve the FEM problem (scalar or vector potential), the force is calculated either by $\mathbf{F}=\partial W'/\partial \mathbf{s}$ at constant current or $\mathbf{F}=-\partial W/\partial \mathbf{s}$ at constant flux.

1) Scalar potential formulation: The magnetic field, in the scalar potential formulation, is defined as $\boldsymbol{H} = -\,\boldsymbol{grad}\ \psi$. The magnetic force is then calculated with the co-energy which, for a permanent magnet, is given by [2][7]:

$$W' = \frac{\mu_0}{2} \int_V (\boldsymbol{H}+\boldsymbol{M})(\boldsymbol{H}+\boldsymbol{M})\, dV \tag{10}$$

The magnetic force on a direction i on a node k is [2]:

$$f_{ik} = \frac{\mu_0}{2} \sum_{e_k} \int_V \left[\frac{\partial H}{\partial s_i}(\boldsymbol{H}+\boldsymbol{M}) + (\boldsymbol{H}+\boldsymbol{M})\frac{\partial H}{\partial s_i} + (\boldsymbol{H}+\boldsymbol{M})(\boldsymbol{H}+\boldsymbol{M})|G|^{-1}\frac{\partial |G|}{\partial s_i} \right] dV \tag{11}$$

Where e_k refers to the elements which have the node k in common, $|G|$ is the determinant of the jacobian matrix. The global force is given by the summation of the forces on the nodes of the magnet. During the virtual displacement, the scalar potential ψ is kept constant [1].

2) Vector potential formulation: In the vector potential formulation, the magnetic induction is given by $\boldsymbol{B} = \boldsymbol{rot}\ \boldsymbol{A}$. The magnetic force is calculated with the magnetic energy, which in a permanent magnet is defined as [2][7]:

$$W = \frac{1}{2\mu_0} \int_V (\boldsymbol{B}-\boldsymbol{B_r})(\boldsymbol{B}-\boldsymbol{B_r})\, dV \tag{12}$$

The force on a node k on a direction i is then [2]:

$$f_{ik} = -\frac{1}{2\mu_0} \sum_{e_k} \int_V \left[\frac{\partial \boldsymbol{B}}{\partial s_i}(\boldsymbol{B}-\boldsymbol{B_r}) + (\boldsymbol{B}-\boldsymbol{B_r})\frac{\partial \boldsymbol{B}}{\partial s_i} + (\boldsymbol{B}-\boldsymbol{B_r})(\boldsymbol{B}-\boldsymbol{B_r})|G|^{-1}\frac{\partial |G|}{\partial s_i} \right] dV \tag{13}$$

The derivation with respect to the virtual displacement is done with constant vector potential $\boldsymbol{A}$.

III. About the Force Distributions

A different force distribution on permanent magnets can be obtained using each of the methods described above. As it has been said, some of these methods are based on equivalent or mathematical models and the resulting force densities represent no physical reality. The aim of the present discussion is to clarify these points and propose a solution for the calculation of force densities on rigid permanent magnets.

The geometry of the problem, used for comparison is presented on Fig. 1.

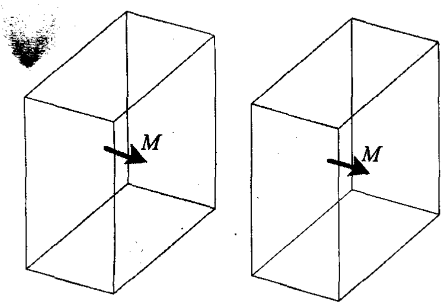

Fig. 1. Geometry used for the comparison of the distributions of forces

Fig. 2 presents the force distribution given by the methods based on the equivalent source and on Maxwell stress tensor.

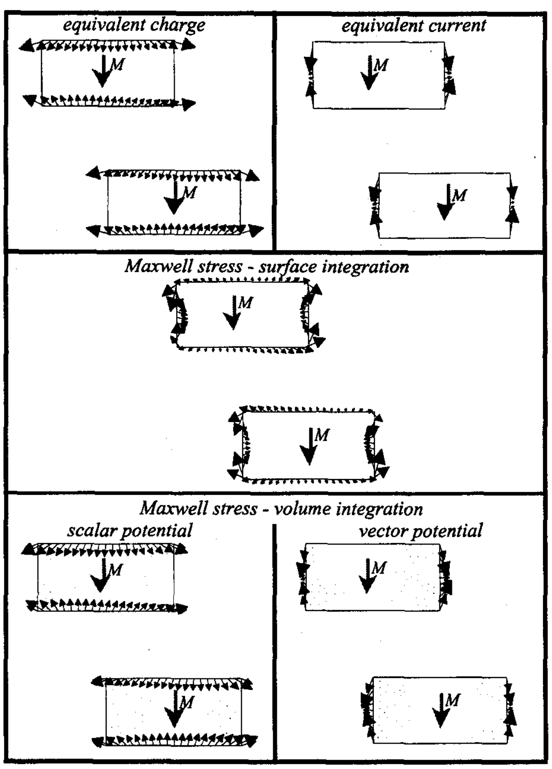

Fig. 2. Forces distributions by the equivalent models and Maxwell stress tensor (no physical reality)

As it has been said before, these distributions do not have any physical meaning. Each method gives a particular distribution and it seems clear that they do not represent the actual force distribution on the magnets. The methods based on equivalent sources and on the Maxwell stress with volume integration give similar results but this is not meaningful.

One must take care with the use of these methods if it is not only to obtain the global force. For the calculation of the force distribution between the magnets, one must not rely on any equivalent or mathematical model but rather on a physical principle such as the derivation of energies.

However, the force distribution obtained will depend on the expressions of these energy or co-energy. For the correct calculation of force densities, a physical analysis has led to decompose the total energy in a magnet in two terms: the intrinsic and the interaction energy. For each one of these terms, corresponding force densities appear [2].

The intrinsic magnetic energy is obtained once and for all during the non-linear magnetising process. This energy and consequently the intrinsic force densities are yet unknown [2][7]. The expressions currently used to calculate the energy assume a linear rigid model for the magnet and give an energy from the remanence to the working point, so, they do not represent correctly the energy implied in the non linear magnetising process [2][7]. The intrinsic energy, and so the real intrinsic force densities, cannot be obtained by such simple models [2]. By these expressions, the (co) energy that is taken into account is the (co) energy from the remanence $\boldsymbol{B_r}$ (or $\boldsymbol{H_c}$) to the working point $\boldsymbol{B}$ (or $\boldsymbol{H}$). This energy includes the interaction energy and the energy due to the magnet alone in the air, i.e., the point that is taken as reference is the remanence one. This energy is called the total energy and the corresponding force distribution is presented on Fig. 3.

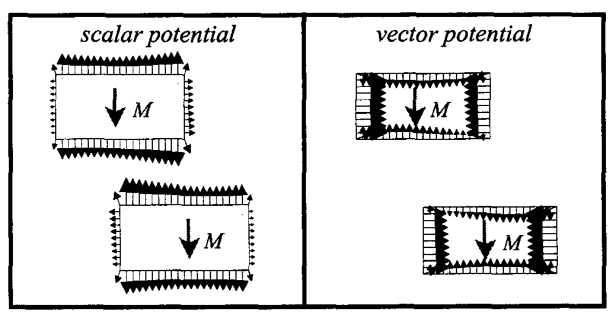

Fig. 3. Total force distribution between two magnets

This total force includes the interaction force between the magnets and the force distribution due to the magnet alone in the air (which is not correctly calculated). This distribution is presented on Fig. 4.

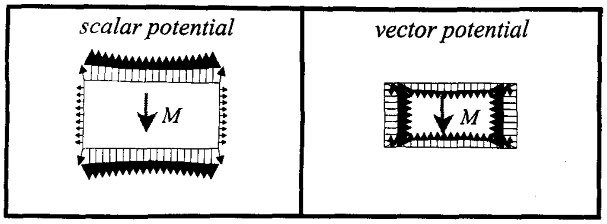

Fig. 4. Force distribution for a magnet alone in the air (virtual work method)

However, when the force distribution is needed, normally, we are not interested on the intrinsic distribution, but on the interaction force distribution between the magnet and the exterior media. For the calculation of the interaction force, one must not take the remanence as reference. The energy and consequently the distribution of forces for the magnet alone are not correctly calculated [7].

For the two magnets, the reference for the interaction energy and corresponding force distribution must be taken from the point for the magnet alone ($\boldsymbol{B_{air}}$) and not at $\boldsymbol{B_r}$ as the formulations assume (Fig. 5).

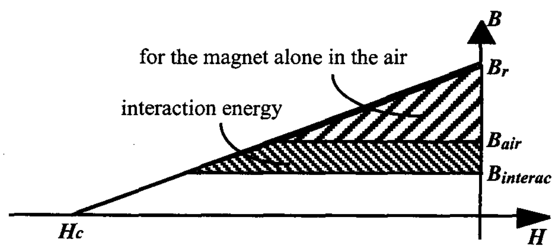

Fig. 5. Variation of energy in a magnet

To obtain the interaction force densities, one must withdraw the force densities due to the magnet alone from the total force densities. That way, the resulting interaction force distribution between the two magnets is obtained and presented on Fig. 6. The reference point is the magnet alone in the air and the drawn forces are the differential forces due to the surrounding media only.

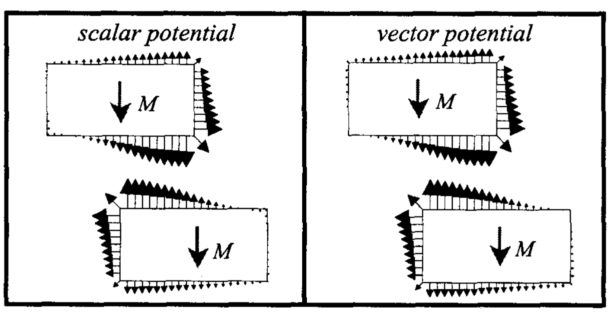

Fig. 6. Interaction force densities between two magnets obtained with the virtual work method

As expected there is only one interaction force distribution between the two magnets. The results do not depend on the formulation and the methods can be used to calculate the interaction force between a permanent magnet and any other exterior medium [2].

IV. Conclusion

Different methods for the calculation of density forces in permanent magnets have been presented and compared. It has been shown that the results given by each method are quite different. One must take care with the use of the methods based on equivalent or mathematical models. They can be only used for the calculation of the global force. But the resulting force densities have no physical meaning.

Nevertheless, the use of the virtual work method leads to coherent results and after some physical considerations about the energies in the magnet, the same distribution of forces is obtained by two different formulations. Thus, it appears that the only method that can be used to calculate the actual force distribution between a magnet and the exterior medium is the one based on the virtual work principle.

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